TRANSCRIPT

Webinar: Building Educators' Understanding of Early Mathematics to Promote Students' Later Mathematics Success

October 22, 2015

[**Slide:** *Building Educators' Understanding of Early Mathematics to Promote Students' Later Mathematics Success*]

REBECCA PERRY

Hi, everybody. Really nice to have you here this afternoon. I think this is going to be a really fun webinar, so I'm really glad you have joined us today.

[**Slide:** *Presenters* (1 of 3)]

I just want to start with just saying that's me, so you know who's talking. That's what I look like, if you haven't met me in person yet.

[**Slide:** *Goals for the Webinar*]

But what I want to do basically right now is just go over quickly the goals of the webinar. I think you probably saw that previously when you signed up, but our goal today is really to jump in to some practice-based recommendations from research, talk a little bit about how this research relates to the Common Core State Standards, particularly for grades K–2. And then, talk a little bit more about how both of these really support mathematics learning in middle and high school.

[**Slide:** *Agenda*]

What we're going to start with is Dr. Douglas Frye is going to walk us through the IES Practice Guide. You're going to hear a little bit about evidence from research in ways you might not have heard it before. After that, we have the pleasure of hearing from Dr. Akihiko Takahashi, who's going to help us make the connection to the Common Core State Standards and provide some practical suggestions for classroom instruction. Unfortunately, Akihiko was unable to be with us here today, so we've added video of his September presentation on this topic into 3 short clips. I'll be introducing the videos and elaborating on just a few points in each of those. And after we're done with the video, we'll have some opportunity for a question and answer with Dr. Frye. And then we'll close with a brief survey, so you can tell us your thoughts on this event.

[**Slide:** *Presenters* (2 of 3)]

So right now I just want to introduce Dr. Frye. Douglas is an associate professor at the University of Pennsylvania's Graduate School of Education. And he's the director of the Interdisciplinary Studies in Human Development program. He was a lead developer on the 2013 Institute of Education Sciences, What Works Clearinghouse Educator's Practice Guide, *Teaching Math to Young Children*. And his research efforts focused on cognitive development in children's early math development and the developmental sequence of early math reasoning skills and activities to support young children's development. So with that, I'm just going to turn it over to Douglas.

[**Slide:** *Using the* Teaching Math to Young Children *Practice Guide*]

DOUGLAS FRYE

Thank you, Rebecca. I appreciate that introduction and I'm happy to be able to talk to the webinar. What I'd like to talk about today is the Practice Guide that we wrote for early math. It's a benefit to be able to talk with you so that you can not only look at the guide, but also hear a bit about the thinking that went into the guide and the actual process that we followed in order to form the guide and write it. The guide is sponsored by the Institute of Educational Studies and the U.S. Department of Education and issued through the IES What Works Clearinghouse. The guide obviously focuses on early math. That means that we concerned with ourselves with children from three to six years of age.

The guide is meant for both preschools and introduction of early math and building of early math in preschools, but also for use by families. So the attempt has been made to both review the research on early math to see what it can tell us about the practices we should follow, but also then give suggestions for particular techniques and approaches that can be used as well as identifying roadblocks and common problems in introducing early math. So let's delve into the subject.

[**Slide:** *Need for Early Math instruction*]

One thing I should say is that there's a clear need for more instruction in early math. If we look at the international studies of early math learning, those are usually at later ages, but even the ones that show how children are doing at the beginning of school, compared to other countries, the U.S. doesn't perform…children don't perform well in math at the beginning of school. So we're certainly not at the top of the list of countries in terms of our children's math preparation at the beginning of school.

Also, if you look at the studies that have been done within this country—within the United States—we know that there's a lot of variations in math knowledge with young children as they're about ready to enter into school. Children's math preparation differs from different families. It differs from whether they've gone to preschool or not. It differs by their socioeconomic level. So we know that there's a lot of differences in how well children are actually prepared for math instruction at school entry.

On top of all of that, though, there's recent research—and I'm citing an article by Duncan here—that shows that that preparation the children have at the beginning of school in early math is actually correlated with their math achievement in third grade. And what's more, it's correlated with their reading achievement in third grade. So young children who are having some difficulties with math at the beginning of school are likely to show those same difficulties, and the difficulties may even show up in something else like literacy. So we think there's a clear need that we do something about this issue.

[**Slide:** *Increased Attention to Early Math*]

There is a great deal of work going on in looking at children's early math understanding. And there are a number of new curricula that have been developed for both preschool and kindergarten. Those curricula include both packages that are directed solely to early math as well as comprehensive curricula that deal with things besides math, math and literacy. We've tried to review the curricula that were available at the time we were doing the Practice Guide, and you'll find the names of some of the most popular curricula in the appendix to the Practice Guide. In Appendix D-2, those are the solely math curricula. And then in Appendix D-3, you'll see the comprehensive curricula that cover more than early math. And we give some background to each of these curriculum and also whether they've been tested and evaluated or not. So the guide may be able to help you find curricula for early math if you want to try to use these curricula that are on the market now.

The Practice Guide itself had a slightly different goal. The idea of the Practice Guide was to try to evaluate the evidence—the actual research evidence—for the different practices that are being followed today in early math instruction. So we were trying to look for the specific practices that have been used to teach particular things in early math and then really see what the evidence shows for whether those practices are effective and how they can be used.

[**Slide:** *Institute of Education Practice Guides* (1 of 2)]

I should tell you a little bit about the Practice Guides, just to give you an idea of how these things are written and what sort of information goes into them because it may help you evaluate this particular one. So the Practice Guides are offered by IES across a whole variety of different topics. And there's a procedure that's followed in each Practice Guide in order to look at the evidence in a given area. And what we're really aiming for in the Practice Guide is to find evidence-based practices—so practices that we know work, that have evidence for them—and then share those practices with practitioners who can actually put them into operation.

How this is done is—the first step is that the practices are collected from the scientific research literature. So there's a review of all the studies we can find on early math and looking at specific practices. And the practices are also suggested and then evaluated as well by a panel who were formed to look at this area. This panel was made up of both researchers and practitioners.

[**Slide:** *Panel Members*]

These are the panel members, and they deserve the credit for the majority of the work on this particular Practice Guide. The panel members include three people who are early math researchers and have spent a number of years looking at different developments in early math. There were two practitioners who were essentially expert teachers. Both of the practitioners on this particular panel also had experience in research and in looking at different instructional practices. Finally, there was one person on the panel who was the methodologist, who was in charge of looking at how the research studies we were considering were evaluated and combined to include them in the Practice Guide.

[**Slide:** *Institute of Education Practice Guides* (2 of 2)]

So as I said—again, just to give you the background—the studies we've looked at were drawn from the research literature. And they were evaluated against a particular procedure that What Works Clearinghouse has standards for. We actually looked at over 2,000 studies in early math and then had these studies evaluated. The criteria that the studies were evaluated against were whether the particular practice described in the study had been shown to have a positive effect. Did it have a positive outcome in changing young children's early math understanding? And then there are quite strict criteria for evaluating what's a positive effect. We're really looking for a causal statement that the practice itself led to a better outcome.

And then the second criteria that was used was, how general was the outcome for different settings and different groups of children? In other words, was the practice tested for only one group of children—a selected group of children—or has the practice been tested across different children in different locations with different backgrounds, so that we can make broad statements about what's an effective practice and what's not. So those are the requirements that What Works Clearinghouse uses and the ones we had adopted for this particular Practice Guide.

[**Slide:** *Teaching Math to Young Children (3-6 years) Recommendations*]

Having gone through that procedure and looking at over 2,000 studies, the panel came up with these five recommendations that we were evaluating for teaching early math to three to six year olds. And I'll just run through these quickly so you can see what they were. Recommendation one is to teach number and operations in early math using a developmental progression. Now most of the research in this area in early math is on children's understanding of number and children's understanding of number operations. So there was a great deal of research to evaluate here and a number of very good studies to be able to see whether a given practice or a given approach made a difference for number understanding and operations.

On top of that, recommendation two looks at other areas in early math. As it's listed in the recommendation, you can see, we should also be teaching geometry or shapes, patterns, measurement, and data analysis or graphing, using a development progression for young children. These other areas of math have much less research available on them. So they're generally seen as being important. They obviously have connections to later areas in math education, but in terms of early childhood, there may not be quite as much research on each of these other areas as compared to the research on children's understanding of number and number operations.

Then there are three more recommendations that the committee could find support for in looking at the literature. Recommendation three, as you can see, is use progress monitoring to ensure that math instruction builds on what each child knows. I'll talk more about progress monitoring and developmental progression a bit later in the webinar, but this recommendation is saying, "We need to try to follow how young children are learning in early math and be able to keep track of where they are in the developmental progression." So progress monitoring helps to see how individual children are moving through developmental progression that the research has identified for early math.

Recommendation four is teach children to view and describe their world mathematically. And the Practice Guide has specific suggestions and techniques about how that should be done effectively. So again, it's something we're encouraging—these techniques—we're encouraging to be adopted into early math classrooms in preschool in order to begin to increase the amount of attention and talk that is given to early math.

And finally, recommendation five is a recommendation that we believe, as a panel at least, that there should be time—particularly in preschool—each day devoted to teaching early math. There should be a dedicated time devoted to early math in preschool. And then we can also suggest that early math instruction be integrated into the other activities that happen during the preschool day or nursery school. I'll give a little bit more information about that in just a moment.

[**Slide:** *Ratings of Evidence for the Recommendations*]

I should say the truth in advertising here. This…these are the ratings given to the recommendations that the panel found for this Practice Guide. These ratings are actually ratings of the evidence in support of each of the recommendations. And as you can see, the ratings more or less reflect how much research is available in each of these different areas. As I said, there's a great deal of research in both developmental psychology and in education on children's understanding and learning of early number. So these levels of evidence for that recommendation are moderate. They would probably even be strong evidence of that recommendation if there were more studies that had a broader sampling of children in them, but, as I say, there's certainly adequate studies to reach a moderate recommendation of evidence.

Then, as you see with the four other recommendations, the evidence is not as available for judging these recommendations. There is evidence for them, or else they wouldn't be in the guide, but this rating scale is saying for recommendations to be able to evaluate enough evidence to make sure that they really are effective. And I should just note that one of the problems there is often some of these different techniques are really built into a curriculum and not presented alone. So they may be presented as something like being able to talk about math with young children. That technique may be incorporated along with using activities with children for math. And as a consequence, testing these particular activities individually has not been done. They've been bundled or packaged together with other approaches in the curricula, so it makes it difficult to be able to pull out the evidence for each of the recommendations individually.

[**Slide:** *Organization of the Recommendations*]

You can see…I think you've probably already noticed, that the organization of the recommendations are into two types. The first two recommendations really talk about early math content areas—what should be taught, what should be covered in preschool and in families before school in terms of the early math area. And we've talked about those and I'll talk a little bit more about them in a minute. And then the final three recommendations really explain more what techniques should be used—what instructional techniques—or what approaches should be used in terms of effectively presenting early math to preschool children.

[**Slide:** *Integrating Math into Preschool* (1 of 2)]

In fact, what I'm going to do is start from the end and go backwards. So I'd like to start with recommendations four and five; to begin with those. And I'd like to recommend in this regard Dr. Sharon Carver's Live Math Daily webinar, which I'll give you the URL to on the next slide. Dr. Carver was on our panel and has produced a webinar that describes how to make practices that conform to recommendations four or five, giving examples of those practices. So it's a very good place to look for specific techniques that can be used in the preschool classroom. I should just say that, as you begin to think about it, there are a number of opportunities—a number of places in preschool and nursery school—where it's possible to begin to emphasize young children's math learning.

The point here of counting pattern sets can be introduced into the preschool classroom easily. Some of the examples that are talked about in the guide and in Dr. Carver's presentation are things like when children line up to leave the classroom or come into the classroom. It would be easy to have children count themselves when they're lining up. It's a natural opportunity for that. So it's easy to incorporate counting into things that are going on in classroom already, that are done every day in the classroom. Another example is in setting up for snacks, putting out different juice boxes on different tables, making sure there are four juice boxes on each table before snack. Again, it makes it possible to take classroom routines and introduce math into those routines along with patterns, so that there can be practice in the early math area as a part of everyday routines in the classroom.

The second point here that the panel and the Practice Guide looked at a number of early childhood books and look at them for their importance, both for literacy and for numeracy. And there are a number of good books that we found that combine the two so that it's almost a two-for-one experience that the literacy can be emphasized in the book and at the same time there is numeracy that's a part of the story. And that numeracy can then be emphasized at the same time. And we list a number of books that we think do that well.

The third point is that there's research now showing that games for young children, specifically board games that include counting, actually are effective in improving children's early math skills. So that's another area that there is good evidence for using board games as a approach to improving children's early math. And again, those are listed and examples given in the Practice Guide.

[**Slide:** *Integrating Math into Preschool* (2 of 2)]

There's a specific technique that the panel found through looking at all of this research for trying to increase the talk about early math in the classroom and for making it possible to have conversations about early math with young children so that it's possible to build on their own math knowledge. So, this technique—just to describe it briefly—is to use the children's own talk about math, which does happen. So when they talk about shapes or when they talk about patterns, when they talk about numbers or when they say something like, "I want more," or "I have more than someone else has," those math statements ought to be noticed and used to start a conversation.

Then the next step in this technique is to try to use open-ended questions that work off of the child's own math comment. And the open-ended math question can start a conversation about what the child has said. And they also suggest multiple representations and solutions. The idea here is that if the child's thinking about a particular situation—a particular math situation—and has one way to think about it or one way to solve a problem of, say, how to make the number of things one child has the same as the number of things another child has, it's good to talk about that one solution to the problem, but it's also helpful to try to propose other ways to reach the same solution. So other ways to solve the problem that the child's working on, and to use other representations. By multiple representations here, we just mean actually using objects, using drawings of objects, using sets, eventually using numbers; but if you can use multiple representations for a problem, you begin to get the child to see how these different representations are equivalent and make it possible for the child to move from one representation to the other.

And, naturally, the third step in this technique, this is recommending that we move from the informal math language the child may be using and if they're comfortable with that language, then move towards the more formal language that they'll eventually hear in primary school. So if they're using the word "and" for putting two sets or two groups of objects together or using "more" for how to increase the size of the sets, that's fine and, clearly, meaningful. And when the child does know the meaning of those words and use them correctly, then eventually it'll be useful to introduce the word "add" as the more formal, technical math word to describe that operation. Again, as I said, Dr. Carver covers a number of these examples and shows how to employ these techniques in the webinar that you see on your screen there. And I invite you to take a look at it for a more complete description of those last two recommendations.

[**Slide:** *What Early Math Should be Taught?*]

Now let's turn back and go to the beginning of the recommendation list. And maybe we can talk a bit about the actual early math content and what we're proposing that young children should have experience with in learning early math. As I said before, the first recommendation is for number basically, and the second recommendation are for some other areas of math that are equally important, but don't have quite as much research about them.

[**Slide:** *Common Core Standards for Kindergarten Math*]

I would like to say that before we do, there's good alignment between the recommendations in the Practice Guide and the Common Core Standards for kindergarten math. I'd like to say that we started out trying to make that alignment, but in truth, I think we started work on the Practice Guide before the Common Core Standards were available. It turned out at the end, when we put the two next to each other, that there was good alignment between our recommendations and the Common Core Standards for early math. Basically, our first recommendation, talking about counting and operations in the developmental progression, corresponds to counting in cardinality, operations, and number in operations, and base 10 eventually. So those first recommendations listed in the Common Core Standards.

The second recommendation, where we pay attention to the other areas of math that maybe has slightly less research on them, correspond to the Common Core measurement and data in geometry. Common Core doesn't talk specifically about patterns, but we think that's important. And in some ways, it's related to geometry as well. So there's good coverage between the two, and work in early math and improvement in early math should match up to what the Common Core Standards are for kindergarten.

[**Slide:** *What Do Children Have to Learn?*]

Let me talk then, while we have the time, about the content that's covered in the Practice Guide. So what do children have to learn in early math? And just to give an example to start with, as an exercise, I'm asking you to please memorize the words on the page that don't look like English words, so those words that are underneath the "Please memorize this list." If you could just, for a moment, look at that list and try to imagine memorizing it, learning it so that you can repeat the list back. We may not actually be able to do this here, but as a thought experiment, you can…you might see what it would be like to try to learn that whole list of unfamiliar words. And then I want you to think, if you could, about what you've learned. If you really could repeat that list from memory, what have you actually learned? You've memorized the list of counting words 1 through 10 in the Klingon language from Star Trek. So supposedly, this is how Klingons count, and these are the number words they use.

So again, if you memorized this list without knowing that, you really don't know what to do with the list or how it can be used. If I tell you that it's a list of counting words then you do, as adults, know what to do with the list now. But try to imagine young children who are learning for the first time the list of counting words in English: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. That's obviously a challenging list to learn. They have to learn 10 things, in order, and be able to reproduce them in the correct order each time. And even if they've done that, even if they've memorized those count words, have they really learned to count yet? And it turns out—what the research shows is there's actually quite a bit beyond just the count words that children have to learn to learn how to count. And it takes them an amount of time—sometimes over years—to be able to master counting in a way that will be useful for later math.

[**Slide:** *Counting and Cardinality*]

Let's look at what else they have to learn beyond just the list of count words—whether in Klingon or in English—in order to be able to count. This little illustration shows what's actually involved in counting. Not only do you have to know 1 through 10, but you also have to be able to indicate objects or whatever it is you're counting and associate a count word with an object. So to count these three stars, we have to be able to point to each in turn and go one, two, three. So not only do we have to indicate each object, but we have to keep one-to-one correspondence between the count words that we've learned and the objects. And when we can do that—when we have a list of counting words and we can apply them to the number of things that we're trying to count, where we use one and only count word per each thing—then we have a procedure for counting accurately, one that obviously depends on one-to-one correspondence between count words and objects.

There's one more step in this that children have to acquire…young children have to acquire. And that's the recognition that there's something special about the last count word that's used. When you're counting one to three objects, that "three" is not only a count word, but it's also a cardinality word. It's telling you as you're counting how many objects there are in the set, what's the total number of things in the set. So the "three" is serving two purposes, both as counting and also as cardinality for saying how many things are in the set. So, as you can see, when young children are acquiring counting, something that looks to us to be very simple to do—easy for all of us—they're actually acquiring quite a lot of knowledge, and it's both verbal knowledge and these principles of correspondence, one-to-one correspondence, and cardinality.

[**Slide:** *Developmental Progression of Number Understanding*]

So I'm using that to—and I'll go into more detail about this in just a minute, but just to give you a look at what's in the Practice Guide—what I've told you about counting in cardinality actually fits into a developmental progression. And this is the characterization of developmental progression that's in the Practice Guide. From the research on how young children learn and develop these skills, we're able to list the steps that children go through in acquiring something like counting and then the more complicated forms of counting. So this is a developmental progression for counting.

[**Slide:** *Steps in the Progression*]

The steps in this progression, explicitly from the research, is that young children don't start with counting. Young children start with a procedure that's called subitizing. And subitizing is what we all do if there are a small number of things—two or three things—on a table, and we look at them, we can tell how many things are there without counting. We can just look and know there are two things there or there are three things there. The same is true for young children; even two and three year olds can look at these small collections and be able to say how many. You could ask them a "how many" cardinality question and they'll be able to say "two" or "three" without having to count. So there's actually a procedure that they can use that's available before they can even count.

The next step in the progression is what I illustrated to you before, and that's being able to count collections of objects larger than three things, and actually using the counting words to be able to find out how many things are there. And as I said, there's a lot that goes into this knowledge, but when the child is beginning to count they can count different objects, they can count a set in different orders—you know, from left to right, right to left, come up with the same number—so that they are developing skills that allow them to count correctly and, finally, to understand cardinality.

[**Slide:** *Counting Errors*]

I don't mean to say that that developmental progression is without mistakes. And you should actually expect young children to make mistakes while they're learning to do counting. And these mistakes become important because they show us how children may be approaching this and what help we can give them. So also in the Practice Guide, there are illustrations of some of the known counting errors that preschoolers make. And these will show up in almost all children's first attempts to count. So it's helpful to be able to recognize these as common errors that children make and will eventually, with enough practice, no longer make.

This little chart just shows the types of errors. One is—on the top—called the sequence error; that children might count 1, 2, 3, 6, 10. So that child doesn't yet know the counting words in English or Klingon, but they are skipping out some of the counting words and will need to learn the full list of words before they'll be able to count accurately. There are some words in English that are known to be problems; words like "fifteen" are one of the most common mistakes, or counting words that are left out in children's counting. Perhaps because "fifteen" doesn't sound like "five," whereas "fourteen" does sound like "four." But again, it will be very common to see children leave out one of the count words like fifteen and have to have help learning the correct count sequence.

There are also another set of errors called coordination errors. And those are really errors of coordinating both the count words and the objects. And children may double-count an object; that's the first example of 1, 2, 3, 5, 6 pointing to an object twice. And they may also skip out a count word. So, say, "1," and "2, 3, 4," when there are actually 5 objects there, or they could skip out an object. Again, these are very common errors that can help show where the child is in this developmental progression.

[**Slide:** *Steps in the Progression* (cont.)]

I want just to give you examples from two later steps in this progression because there are more complicated forms of this counting, but when the child can actually learn to count accurately, they're able to compare sets of objects. At the beginning, they can make those comparisons with subitizing. They might be able to look at two things and at three things and without counting say, "there are more for the three things." But when larger collections of objects are given to them, say, 10 things and 8 things, then that will require counting. And it won't be until the child can count accurately that they'll be able to count the cardinal values of each set and compare and say which has more.

And then again, as we're nearing the end of preschool, entering into school age—so now we're talking the five and six year olds—another step in this progression is that eventually children will be able to start counting with any number that they know. They won't have to start with one, but they could start at five and know that the next number that comes after five is six. So it's actually a development to be able to break up the count sequence and start at any point. And actually, that number-after knowledge is valuable for doing addition and is one of the later steps in the number sequence.

[**Slide:** *Similarities in Other Aspects of Math*]

Just let me give you one more example in using those other aspects of math. I'm going to try to make some of the same points that show their application to this other area. The example here that's in the guide is talking about shapes. So what do children learn about shapes, geometry, and the developmental progression there? The first steps in this is that children become able to label or identify simple shapes like what's a triangle or what's a rectangle, what's a square.

What we're recommending as a technique for supporting this knowledge is to use examples with children, but to use examples of things that both are the shape and things that are not the shape. So the research seems to show that negative examples—what isn't the shape—are actually very helpful in defining these categories of what's a triangle or a rectangle. So in showing a child that a triangle has three sides and three points of vertices, it's also…it's useful to say, "This is a triangle," but it's useful to show what isn't a triangle, to show a rectangle, show that that has four sides and that makes it not a triangle. So the negative evidence seems to be very important in trying to show children what the categories actually are and what belongs to them.

And then, as it says, memorization or just learning to label these different shapes is valuable. Knowing what makes a triangle a triangle and a rectangle a rectangle is valuable. But we also found that the evidence seems to show that trying to work with relations between shapes and composition and decomposition with shapes—so the child learns operations—also helps in acquiring what's necessary for geometry.

[**Slide:** *Relations Among Shapes*]

So this is just an example…set of examples…about the sort of combination and decomposition you can do with shapes, similar to what you can do with numbers. Obviously, you can take two squares, combine them together to get a rectangle. You can also cut the rectangle in half and get two squares. The same thing can be done for taking a square, cutting along the diagonal to form two triangles. Those sorts of operations for young children seem to be helpful as well and show what can be done with shapes beyond just identifying particular shapes. So there are a number of examples in the Practice Guide that show how to do combination and decomposition using common shapes that children will be familiar with. So those are two examples, one for number and one from shapes, that show the kind of techniques that we're trying to make available and have evidence for as being useful for children acquiring early math.

[**Slide:** *Value of Developmental Progressions*]

Let me just say a word in the remaining time about why we adopted developmental progressions and what value they have for teaching. It's a feature of math learning in particular that you can find these progressions. You can find an order in which children typically learn these early math skills. Given that that's true, and when can we identify a developmental progression for how children learn in this area, it has real advantages. One thing is, if we know where the child is on the progression, we can actually give instruction at the child's level, so that we can pitch the instruction at the right level for the child's understanding—because, as we know, some of these steps are learned before the more advanced steps.

And as I mentioned before, it's also…if there's developmental progression, possible to use progress monitoring to assess where the children are in the progression. So it's possible to use either little tasks that have been prepared to see where the child actually is or to use the common activities that are being done already in early math to get a sense of where the child is in the developmental progression. And then they use this information from progress managing to be able to say, "This child seems to be at this level," and we can identify instruction approaches and content that are at that level or above to help the child make the next steps in acquiring their understanding of early math.

[**Slide:** *Characteristics of Developmental Progressions*]

Add one more piece to that—these developmental progressions—that we know that there's some necessary steps in them. We know that there's some things in math; to learn those things, there are prerequisites. There are other things you have to know first. So, for example, to eventually know how to do simple addition of five plus three, that requires a knowledge of cardinality. That requires the knowledge that five stands for five things and three stands for three things. And then when you combine those two sets, you'll get eight things. So there are some parts of these developmental progressions that have to follow in order. And again, that's an advantage because we can make sure the prerequisites are learned, the young children have their prerequisites before they moved on to the more advanced activities.

But, unfortunately, it's not quite that simple, it's not quite that straightforward because there's not always a strict order to all of the steps in these progressions, and I want you to be aware of that. There may be some variations in the steps, and it may not be possible to say that each child will go through this exact same order of steps.

[**Slide:** *Comparison Step in the Progression*]

And I've already given you one example of that when I've talked about subitizing and counting. Turns out that children, when they're subitizing and see the numbers of small objects, they can also compare those sets at that time; so they can make a comparison of what three things are being more than two things, and they can do that even before they know how to count. However, when they have to learn how to count, they go through that whole process of learning all of the different skills that go into counting.

And then, finally, they can go back to comparing sets using counting. They can count—if it's a larger set—how many is in one set, the other set, how many is in that set, and say which one is larger. But there's no direct relationship to be able to say, "Children learn counting first and then they learn comparison," because for small sets, they do the comparison at the beginning of the sequence. So these are approximate progressions that we believe are very helpful, but they can't be read as applying to every child for each of the steps. They're more guidelines to what is usually learned before a given later ability and what's learned after that.

[**Slide:** *Conclusion*]

So, I think I'm running out of time…let me just conclude that because math learning is progressive—that the main steps do occur in order—so developmental progression and progress monitoring allows us to get an approximate idea of where children are on the sequence and allows us to arrange instruction. And the early math curricula that are covered in the Practice Guide try to do this; arrange instruction so that it fits the sequence and targets the appropriate level. And the value of this is that if we can find the right techniques to use for early math learning—the ones that are supported by evidence—then we can present the right techniques to…the right skills to the children at the right times, and that this learning will actually make a difference for children's school readiness in early math and their math learning in school.

[**Slide:** *Availability of the Practice Guide*]

And let me end with giving you a place where you can find the Practice Guide. It's free. We hope that you can use it and take advantage of the recommendations and techniques that are illustrated there and begin to introduce more early math into preschool classroom and into families for getting children ready to learn math in school.

REBECCA PERRY

Thank you so much, Doug. It was very nice to hear what you had to say.

[**Slide:** *Presenters* (3 of 3)]

So next, what I'm going to do is just briefly introduce Akihiko Takahashi. Akihiko is an associate professor of math education at DePaul University in Chicago. And at DePaul, he teaches math teaching and learning and mathematics for prospective teachers and practicing teachers. He also provides workshops and seminars for practicing teachers using ideas from U.S. and Asian countries, including immersion programs in Japan to support teachers from other countries to learn about Lesson Study, which is a form of teacher professional development that originated there.

Before moving to the U.S. for his graduate work, Dr. Takahashi was an elementary teacher in Japan for nearly 20 years, teaching self-contained classes in grades 1 through 6. And during his elementary teaching career, he was nationally active in math lesson study and mentored about 200 pre-service teachers before becoming an educator of math teachers. He co-authored the bestselling Japanese elementary textbook series, *Math for Elementary School*, which has been translated into English, pilot tested in grades K-2 in U.S. schools, and shown to have a positive effect on student teaching and learning.

So we asked Dr. Takahashi to focus his comments particularly on primary school students in grades K through 2. And we'll hear many commonalities between what Akihiko has to say and what Douglas has just talked about.

[**Slide:** *Video 1*]

So what I'm going to do is start by showing you Dr. Takahashi's presentation from September on planting the seeds for Common Core State Standards. And in this first video, Dr. Takahashi emphasizes the importance of problem solving.

AKIHIKO TAKAHASHI

I'm going to talk more on the K through 2 Practice Guide related to Common Core State Standards and how this type of Japanese curriculum materials support students to learn mathematics. So the Common Core State Standards, the states promised to implement in the fall of 2014; however, in some of the places it is not started yet. And one of the reasons is they do not have curriculum material; it's very difficult. So I always recommend, because you do not have a good curriculum yet, why don't you start with mathematical practice, so which you can implement in many different ways, using different…but just looking different in the existing textbook and then teach in different ways. So, that is my recommendation.

And then, as you know, that the Common Core State Standards mathematical practice number 1 is problem solving. So the problem solving is a big emphasis, and which is not new; since '80s. NCTM released An Agenda for Action in 1980s. They said '80s should be the decade for problem solving. So, by the way, what is the definition of problem solving? It's not just simply solving problems. So in the NCTM standards, problem solving means engaging a task for which the solution is not known in advance. That means if students knew how to solve a similar problem, we do not call it problem solving. We have a different name for it. Do you know how we call it? That's called "exercise." There is a subtle difference between exercise and problem solving, so that we should be able to…we encourage students to attack problem which students have never seen before, by themselves, and then students can use their prior knowledge to solve this problem. That is problem solving.

Sometimes we discourage…for example, like many students say, "Teacher, I have not learned this problem, tell me what I should do." Well, that is evidence of the lack of problem-solving skill, right? So we don't want to have such kind of students. Even if we give a problem students have never seen before, students say "I can do this," and then solve. That is perseverance, which is a big issue because it's not so common in the United States, even though that was emphasized since 1980s. "Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from the students' lives." This is a quote from the NCTM standards.

So what this means is, you want to give a problem which students have never seen before in the everyday lesson, in the everyday mathematics, right? And then through this process they struggle, productive struggle, which is kind of a buzzword; everybody is talking about it nowadays. Through the struggle, students should be able to learn something new. That means just solving a problem is not the goal; finding answer is not the goal. By finding solution, by finding answer, you want to learn something new. So the teacher's job is to give a challenging problem for students—not too difficult, not too easy, right amount of challengingness. Then, let them struggle and then bring these struggles to help students learn something new. In order to do so, students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns. That—strategy is very important. Strategy—if students have a strategy, you could solve it, and, moreover, the students should have multiple strategies, wide variety of strategies. Why? If one approach/strategy doesn't work, you don't want to give up. Mathematics is teach a way of thinking. If you teach more, if you teach for test, you never get more than test, more than you study, right?

Of course, you can teach more and you can get more, but it's kind of cramming. However, if you teach fewer but students can [do] more, that is something we want. That is Common Core. The Common Core is fewer topics, right? And then you spend more time and really understand so that you can be ready for wider variety of problems, including something they have never seen before.

[**Slide:** *Video 1*]

REBECCA PERRY

So in his presentation in September, Dr. Takahashi emphasized that developing the standard for math practice should begin in kindergarten and first grade.

[**Slide:** *Developing these practices begins early*]

In the next video, Dr. Takahashi offers these four recommendations—developed from studying both Japanese and U.S. math standards—for how educators can nurture students' development of the standards for math practice. He then elaborates with examples on his first recommendation, present problems in understandable context. And that's very familiar to what Douglas was talking about earlier, too.

AKIHIKO TAKAHASHI

So, presenting a problem. So, present the problem, understand the concept. This is like using tools, modeling, reasoning abstractly and quantitatively, which is related to mathematical practice. Here: 5+3=8. I see lot of worksheets just like this one, right? And asking students to write 8. First of all, this is not a mathematical expression, and this is not a common language. This is just you can jot down with paper and pencil, but it does not communicate very well because country by country, internationally, it's very different. However, mathematical expression like "equation" is universal.

So this is the picture, which is concrete. How many fish will there be altogether? So we have one, two, three, four, five and then three is about to join. So if you show this one in the block, like semi-concrete like this: one, two, three, four, five, and then three moving, in action—Students say, “Oh, this is addition because increase something,” an add-to situation, right, in the Common Core. So this you can write in 5+3=8, and then five fish. Concrete, semi-concrete, abstract, and then bring back to concrete. So this is kind of the modeling process—start from concrete and then model with the manipulatives and then model with mathematical expression, and then do the operation and then once we have got 8, bring it back to the original situation; bring back to concrete.

So another one; concrete situation is cats: one, two, three, four, five cats, and then if you go to semi-concrete, it doesn’t matter if fish or cat, it’s the same, right? And also the abstract is the same. However, five cats, the answer is different. So this process is mathematics. Start with concrete, then do the abstract process and then coming back to the concrete. And then sometimes they skip; students write this one and then 5+3=8. So in that case, students say, “I don’t need blocks.” But how this child can show 5+3=8 is reasonable? If you ask students, they may say, “Oh, that’s something I have.” But we should challenge students: “Are you sure?” So how students should do that? “Let me count.” That means let me go back to concrete and then count picture, or let me show this is addition because they are going to use this one. So it’s not always just one way. Students should be able to go back and forth. So that you could justify their solution, which is important part of reasoning, right—reasoning abstractly. This is a mathematical…one of the mathematical practices.

In order to do so, as a teacher, we have to stop telling the student, “You are right,” or “You are wrong.” That is something we expect students to do. That is reasoning. Justifying their solution is an important part of mathematical practice. Traditional classroom, we as a teacher, we always justify, but if teachers keep justifying, students heavily rely upon teacher.

[**Slide:** *Recommendation #2*]

REBECCA PERRY

In his presentation, Dr. Takahashi’s second recommendation is give students opportunities to attack open-ended problems so the students increase their confidence. We aren’t going to show video of this part of his presentation, but I want to describe what he means by this with a few of his examples. And again, you’ll hear similarities to what Douglas talked about earlier.

[**Slide:** *10. Make story problems for the math sentence 5+3*]

Take a look at this page from the Japanese mathematics textbook. And on the top of that, if you can’t see it, it’s “make a story problem for the math sentence five plus three.” And you’ll see students doing various different things at the park here. Dr. Takahashi describes how problems like this one encourage students to develop their own math problems based on the different elements of the picture, by focusing on the birds or the trees or the students jumping rope. He argues that it is important for students to do their own thinking. Then the teacher’s job is to help students understand these different possible solutions and see the similarities and differences between them by comparing and discussing them.

[**Slide:** *Problem*]

The process of comparing and discussing different solutions is called *neriage* in Japanese. And it's a very important part of Japanese mathematics instruction. Problems like this and neriage give students opportunities to construct a viable argument and critique their reasoning of others, which is one of the math practice standards. So here's a diagram that illustrates the process of neriage, which shows various different solutions coming up from students that comparing and discussing process. And that leads to new ideas and questions, or problems that students might have.

[**Slide:** *7. Make story problems for the math sentence 7–2*]

Well, here's the second example that supports students' understanding of subtraction. So here is, "make a story problem for the math sentence seven minus two." The picture could enable students to show two different kinds of subtraction. So, for example, there are seven trees, two are rounder trees, so how many Christmas tree-like trees are there? There's seven minus two. Another way to think of that is to look at the benches. There are nine total benches, but the problem's not nine minus two. There are seven blue benches and two white benches. So the question is what's the difference? So you can see how this picture could enable students to come up with many different kinds of story problems. And in fact, some students—excuse me—some students might not even use this picture to develop their own story problems for seven minus two.

[**Slide:** *Video 3*]

REBECCA PERRY

So we're going to watch the third video now. And just to let you know, there's a section of the video toward the end that asks you to think about how to use the making sense strategy for the problem seven plus eight. We're not going to have time to stop and do that, so I'm just telling you about that problem right now so you'll understand that part of the video.

AKIHIKO TAKAHASHI

So this is KOA that is in the kindergarten, which talks about cardinality. And then help students to see that the variety of…like if it's seven, is this five and two, or three and four, one and six, rather than just ordinality. If you only know that the ordinal number, the seven, is after six and before eight, that's it. However, if you see this subitizing, and then also the cardinality really understand, you can decompose numbers. So the kindergarten, that is something you want to help students to do a lot of activities and games and then [become] fully comfortable to compose, decompose numbers up to 10.

So if students have a good sense, number sense, rich number sense, addition up to 10 shouldn't be a problem. They can automatize this process. If you look at 7+3: "Oh, I understand that 10 is 7 and 3 so this is going to be 10." What we should do in the first grade? 1.OA.6—Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten, decomposing number leading to ten, using the relationship between addition and subtraction knowing this…blah, blah, blah. So actually, Common Core expects students to be able to think mathematically to find a solution, rather than simply memorizing.

So, for example, again, let's begin with like a concrete situation. Yuka (it's a Japanese kid's name)…and Yuka collected nine acorns and then Hiroshi collected four, how many did they get altogether? Of course, you can count on: 9, 10, 11, 13, 14, 15—it's too much? Kind of like that, you can do this. You can even add up on your fingers, right? But it's tedious. So let's see. Counting every time is hard, isn't it? Can you do it without counting? How can you do that? By using prior knowledge, which is problem solving, right? Of course, you can do by using fingers to count, but without counting, how to find this? Students have never seen it before, right? Let them struggle.

What student might do? If they say, "I have no idea, I cannot do it," that is showing/demonstrating their lack of ability for problem solving. Or some students might say, "Well, I need one more to make 9 to 10," right? So the four is one and three. I'm going to give you 1 to 9 so that it becomes 10. So it's going to be 13. You don't have to count one by one, right? Like this. And some kids say that way. However, some other students have no idea what he is thinking or talking about. So how can you explain? Why don't you bring blocks to help your friend to see what is in your head? So then I did it this way to make 10, right?

[Pause to Practice]

So if you go through this process, if you remember, if you memorize 7+8 is 15—if you forget, forget, right? But if you go through this process, even if you forget, you can recreate this. That is conceptual understanding; you can reproduce. So just memorizing tons of addition tables, that is hard; but to go through this process and then repeat until they automatize, which is different from memorization and automatization.

Memorization—just like nowadays you don't have to memorize any telephone number. Like asking a student to memorize 10 telephone numbers; if you forget, you forget. Even the up to 10, a single-digit addition and subtraction, students should be able to count by fingers; however, it helps students to move away from this one and then helps them to be able to think. In order to do so, kindergarten is so important. For the kindergarten students should be able to compose/decompose numbers very comfortably through concrete activities. So everything is connected. So, therefore, I say, putting a seed. The young, like pre-kindergarten, kindergarten, first grade, is so important. If you do not put a good seed, nothing will grow, right? So you have a huge responsibility putting a good seed. So what is a good seed? Not like a bunch of seeds; it's the important one.

Well, thank you very much.

[**Slide:** *Video 3*]

REBECCA PERRY

So that's the end of our two formal presentations. So what we have now is just a…a little bit of time left for question and answer. We have Dr. Frye here with us live. We have me instead of Dr. Takahashi, so I'm happy to take questions for him now, and I can do my best to answer them or I can certainly relay them to him and have him get back to you later. But if anybody has questions for Dr. Frye, that would be terrific to hear right now. I am going to start with just two questions that have come up for Dr. Frye, and maybe he could start with answering those. And then if the rest of you have additional questions, we can go with those. So, Douglas, I wonder if there's anything specific in the guide about recommendations specific to English language learners? Can you just speak to that point just very quickly?

DOUGLAS FRYE

Yes, I can answer it quickly because I can say that there isn't anything in the guide that does specifically for English language learners. There are, obviously, issues in learning early math from different linguistic perspectives. And, of course, even learning the count words, to learn those in different languages can affect the process, but it's not something that we treated specifically within the guide itself.

REBECCA PERRY

What about your ideas or strategies for helping parents prioritize the ideas that are in the guide? There's a lot of information in there. What should they think about first or second, do you think?

DOUGLAS FRYE

I really think the approach to guide in terms of the developmental progressions and the progress monitoring, it would be best for parents to try to start where the child is. So look at what the child is doing with early math already in either trying to count or trying to compare sets or looking at shapes and trying to classify shapes. So using the child's own interest and then trying to match that to the developmental progression will give a guide to what it would be great to work on next and also what sort of activities or instructional type links would be meaningful at those different steps in the progression.