TRANSCRIPT

Instructional Shifts in the Common Core State Standards for Mathematics

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[Slide: CCSS mathematics]

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. . . sponsored by the National Governors Association along with the Chief State School Officers, and the standards belong to the states separately, and only to the extent that they choose, collaboratively. But the writing was definitely collaborative, and as you probably have heard, 46 states are now signed on, planning to use the assessments that are developed by one of two consortia as their state test beginning in the 14/15 school year, which means California will have, replacing the CST, a test which is being used by 20-some states. And so we're looking forward to that day with some trepidation, so I'm not sure what that test will look like yet, but we'll take a look at some of the items that may be on the test during this talk.

A number of you have heard me speak before, and there will be some redundancy, and I plan to break for questions. I encourage you to ask questions that take us more deeply into the standards.

So to start, we were charged with basing these standards on evidence rather than on politics, which meant our main work was to design standards that helped solve a problem, rather than to commemorate political agreements. This meant that the standard tools of a political process—compromise, consensus building, and so on—were not our main tools. We did some of that but not a lot, certainly not as much as states had to do when they wrote their standards. We were mainly trying to solve the educational problem of U.S. mathematics curriculum, to the extent you can do that with standards.

So what was that problem? What was the evidence? When we looked at research, we looked at really three kinds of research. We looked at evidence from high-performing countries, we looked at research conducted by researchers, and we looked at lessons learned from three decades of standards-based accountability systems. And that's important, because we were *not* trying to find the common ground among the states; we were trying to lead the next step based on lessons learned. And all—okay, and all—so all the evidence from all sources, there was one thing that was predominant, and I'll return to it a number of times. The problem is that the U.S. curriculum is a mile wide and an inch deep. And so that's the problem we designed these standards to help solve, and by the end of this session, I hope you all have a deeper understanding of what mile wide, inch deep means. *We* certainly learned a lot about what it actually means.

So as we get into it, I'm going to first pose a math problem for you.

[Slide: example item from new tests] This item is among the items that the testing consortia put out to the vendors as an example of a certain kind of item that they wanted. Just take a look at what it's asking students to do. This is a little different than what we're used to, especially in California. It's not multiple choice; it's short, constructed response. The kids would enter four fractions. This is a very easy problem for students who understand fraction equivalents, and who understand that a fraction is a number. Both fraction equivalents, and that fractions are a number, are mentioned explicitly in the Common Core standards. And so for students who understand that, this would be an easy item. For other students, this would be very puzzling, and they wouldn't know what to do and they would probably get it wrong. So this is a different kind of assessment strategy than CST has used in the past in most states, in which they have items that are designed to put students on a scale, which is a good idea if you're trying to select students and distribute and differentiate them. A real standards-based test has more items like this, where it's considered good news if everyone gets it right, rather than bad assessment.

[Pause; moving through slides]

[Slide: Problem from elementary to middle school] Okay. So here's another problem, and I'd actually like you to work on this problem, and enter into the chat box your results. Now, most of you are probably by this time wondering, is something wrong? You'll notice this problem doesn't have a question, and more to the point, it doesn't have an answer. So I'm going to give you a little. . .just an assignment. I'm going to ask you to write the question. What question—write a question that makes this information into a word problem. Keep it simple, and enter that question into the chat box. So just write a one-sentence question that uses this information and makes it into a word problem. So I'm looking in the chat box for some questions.

[Pause while receiving responses]

Okay, good. I've always wanted to use silence as part of a webinar experience. So we see a variety of questions up there and they roughly sort into three types. Some of the questions are, "How long did it take to run a given distance, like 20 meters?" Others are of the type, "How far could they run, could Jason run in a given time like one minute or 10 minutes?" And the third is, "How fast is Jason running?"

[Slide: Three kinds of questions can be answered] So these three types of questions—how far in a given time; how long does it take to go a distance, a given distance; how fast is he going—this is the core of the mathematics we want students to understand about rate in sixth and seventh grade. We want them, in fact, to understand how these three questions are related to each other, and these three questions all are three different questions that come from the same mathematical structure, which is expressed in an equation: distance equals rate times time. But understanding how, for that one relationship between distance and time, you get three questions, is a central idea of what proportionality is all about in the Common Core standards in sixth and seventh grade. And you notice we get to that core mathematics fairly quickly by having the students generate the questions. Had I simply asked *one* of these questions and asked you to come to the answer, you could easily skip right by the mathematics and go straight to the answer. This is going to be important as we go on, this idea of how do you go to the mathematics rather than going just to the answer?

[Slide: How do these two fraction items differ?] So here's another item that's in the pool of items the test makers are looking at; it's actually two. The second one which you see there—*which is closer to 1: five-fourths, fourth-fifths, three-quarters, seven-tenths*—that's the kind of question you would have seen on an old-style test. The first question up there—*four-fifths is closer to 1 than five-fourths; show why this is true on the number line*—is more like what you're going to see on the new tests. I want you to think for a second about how these differ. What's the difference in demand that these place on students, these two different kinds of items? And if you have some thoughts, type them into the chat box. If you're with a group, you can talk to each other about it. What's different about what students have to do to respond to these two questions?

So it's clear that in the second one, the action of the student is to select the right answer. In the first one, the student is given the answer; they're told four-fifths is closer to 1 than five-fourths, and what they're responsible for producing is an explanation, an explanation that shows why it's true. And furthermore, they're required to show it on the number line. So this ability to produce an argument, which is what you're doing when you're showing why something is true, and to use the number line, are both explicit in the Common Core standards. That's why this is an appropriate assessment item.

[Slide: Old State Standard] All right, so here I'm going to do a comparison of a couple of standards. This is the old California standard. This is related. . . this is a fifth-grade standard, and it's the standard that covers adding fractions with unlike denominators. So the heading of the section is: *students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals*. And then the subsection that's about fractions: *simple problems including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers,* and so on.

Here is the Common Core standard, fifth grade, related to fractions with unlike denominators. **[Slide: Use equivalent fractions as a strategy to add and subtract fractions]** *Use equivalent fractions as a strategy to add and subtract fractions, and subtract fractions with unlike denominators, by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators*. How is this [Common Core] standard different from this [old state] standard?

I had the interesting experience in Chicago of listening to three Japanese educators explaining the Common Core standards to American teachers. I was sitting in the audience, and one of them, Aki—who's since become a good friend, Akihiko Takahashi—put these two up, and asked the American teachers, "What difference do you see?" They gave their answers, and he said, "You're blind," because their answers amounted to: there is no difference; both standards require adding of fractions with unlike denominators in fifth grade. So to the Americans, who think of standards as topics and standards, answered the question, "At what grade level do you teach which topic?" There is nothing different—same grade level, same topic. What Akihiko pointed out is, these standards say very different things about adding fractions. In the one case, the Common Core, the mathematics, the equivalent is explicit. The target of this standard is equivalent fractions. In the other, the target of the standard, the old California standard, is getting the answer to a fraction problem. This is about getting the answer to a fraction problem [old standard]; this is about understanding the mathematics and using them [in a] mathematically important way [new CCSS standard]. Why equivalence? This is the beginning of algebra. In algebra, we spend most of our time changing expressions into equivalent expressions, changing equations into equivalent equations. Equivalence is enormously central to algebra and all of high school mathematics. And it begins, the equivalence curriculum begins, in fractions. In fact, it's probably fair to say that equivalence is more important than adding fractions. And so we use adding fractions to prepare our kids for the algebra that's ahead of them, rather than teaching some other method for adding fractions that goes nowhere.

[Slide: Old Boxes] I show you this example because, as states and districts are moving towards implementation of the standards, there's a lot of behavior that's just taking the old standards, tossing them out, and swapping in the Common Core standards. If this is all that happens, nothing will really change. These standards were written as a platform for a new kind of instructional system, and I'll be explaining what that is. But I must say that writing standards is the easy part. The hard part is the people part, which is where we are now, and what you're working on. So you're doing the hard part; we did the easy part.

[Slide: Mile wide inch deep: causes, cures] So let's figure it out what mile wide, inch deep means. What the causes are; what the cures are.

[Slide: Mile wide inch deep] First of all, mile wide, inch deep gets across the idea there's too much. The teacher surveys—the teachers said, all across the country, "There's too much to cover"; "I'm moving too fast"; "I have to get it covered by the state test." You've heard it. College faculties, surveyed about what topics were important for incoming freshmen, focused on far fewer topics than high school teachers. So our K–12 system believes covering more topics is important. The university faculties say, no, covering fewer topics in more depth and more solidly is more important. In fact, the Mathematics Association of America, the main organization of mathematicians that deals with education issues, recently issued a statement saying calculus is a college course, and high schools should stop teaching it. This is a statement issued about a month ago.

When you look at a mile wide, inch deep, it's a rate issue. It means too little time per concept. We need to spend more time per concept, which means we need to teach fewer concepts. More time per topic; we need less topics. More time per problem, which means less problems. Our tendency in the U.S., in contrast to high-performing countries, has been more, more, more; faster, faster, faster. That's not the way to do it. Singapore's website, the banner says, "Teach less, learn more." So solving the problem of mile wide, inch deep means focusing on fewer concepts and spending more time on them. Another way to look at this, if you're leading a class trip and you see the kids are too spread out, what do you do? Well, if you have common sense, you stop or you slow down and get them bunched up more coherently, but that's not been the U.S. educational policy stance. We're more focused on striking a handsome posture than we are on solving the problem. And so our response to the fact that other countries are outperforming us is to go faster, to accelerate. Why do you think going faster, when the kids are already too spread out—it just doesn't make sense. And indeed, when you look at variation in achievement, the U.S. is more spread out than other countries. Our kids are more varied in their achievement than other countries, which is a symptom, to my way of thinking, that we're going too fast.

[Slide: Two ways to get less topics] So how do we get to less topics? One is to delete topics, and we took that up as part of our responsibility in writing the standards. And we did drop topics. I'll give you some examples. And we took a lot of flak from the states—our states were our main clients—and what the states told us was, "Hey, you forgot such and such," and we would say, "No, we took it out on purpose," and they would say, "But it's good," and we said, "Yes, we know it's good." We looked at state standards, we looked at what was in the American textbooks, and we didn't see bad stuff. So we realized we're going to have to delete good stuff; it's the only way to get to less. States kind of were uncomfortable with that, but we stuck to it. That was one example of the benefit of doing a problem-solving design process, rather than a political process. Had it been political, we'd have been forced to put those topics back in.

The other way of getting less topics is through coherence. As Aristotle pointed out, every field has a grain size at which it is its simplest, most coherent, and closest to the truth. And in mathematics, that's just one step deeper than we're used to, and it's not as deep as mathematicians would like, but one step deeper. When you go down one step, the coherence pulls topics together so there's actually less to learn. Rather, if you have a hundred different kinds of problems, you could either learn a hundred different kinds of ways of solving them; that's a hundred things to learn. Or you can go one step deeper and see that there's just a dozen principles that can be used in slightly different ways to solve all those hundred problems. And now all you have to learn is a dozen things and how to apply them to a variety of situations. So a deeper understanding, with a little robustness, creates a lot of efficiency. And so we tried to create that coherence across/within a grade level. For example, in third grade we put multiplication, which mainly, multiplication is the most important in the third grade; we put area in third grade because we wanted area to be part of learning multiplication. So area was coherent with multiplication. We didn't put it there because geometry demanded it come in third grade. So there, the arithmetic was dictating what geometry was within a grade level. And we also did coherence in the progression across grades.

And I think this is going to be very important to understand, because it relates directly to a misunderstanding about standards which is prevalent in this country. A progression across grades is about the way in which knowledge and proficiency builds up from grade to grade, grade to grade within a given topic. And I'll talk a little bit more later about how we did that with fractions, to give you a good example.

[Slide: two ways to get less topics] But before I do that I want to just give you an example of what. . .we were urged by some people to drop long division from the curriculum. We looked around and we noticed the high-performing countries, and of those, the ones that were most useful to us were Singapore and Japan. We noticed they kept . . . they still had long division. We actually asked the guy from Singapore. . .asked him why he had long division in the curriculum. The guy from Singapore says, "Oh, well, there's so much unfinished business about place value in fifth grade. We introduced place value already in first grade, and more in second, and more in third, and more in fourth; there's no way that students that young are going to understand something as difficult as place value. And so when they get to fifth grade, most of the students have unfinished learning related to place value. Long division is our way of finishing the teaching of place value, because long division brings place value to the surface. It brings all the unfinished business up to the surface as they try to learn long division. Most of the time while we're teaching long division, what we're doing is finishing teaching place value."

Now, you can almost hear American teachers saying, "How am I supposed to teach long division? They're supposed to know place value before they get to me. How can I teach them long division?" And you can almost hear school districts saying, "This is fifth grade. You're supposed to be teaching long division, you're not supposed to be teaching place value." And you can also hear Americans saying, "If they don't know place value, that's below grade level. Put them into remediation and reteach place value." All these American ways of thinking are what's getting us into trouble and digging the hole deeper, and making the problem hard to solve. Notice what they did in Singapore, and we confirmed they do the same in Japan. Inside a grade-level problem, long division, they opened the window and they could see all the way back to first grade. And they didn't ignore first grade, second grade, and third grade. They treated it as unfinished business. They didn't reteach place value, they didn't set aside long division and reteach place value; not reteaching. What they did was, inside long division they finished teaching place value. And this is what we have to do much more of, and what we have to do less of is reteaching, okay? So, okay, let's see.

Now I'm starting to see a few topics here [*in the chat box*] which—I think I'll address them. The question about "How are the clusters determined?" and "How do we decide what grade levels to teach what?" I'll get to those, but let's get into this mile wide, inch deep even a little further. **[Slide: Why do students have to do math problems?]** *Why do students have to do math problems anyway*? So here's our multiple choice question. I won't ask you to type in A, B, C, and D because we're good at multiple choice; we know D is the correct answer. But when you actually look inside American classrooms, it's as though A is the correct answer, and that's because deep down, I think this is the main problem we have in our curriculum. We're teaching answer-getting instead of mathematics.

[Slide: Why give students problems to solve?] Now, answer-getting is important; it's essential. I'm all for it; I'm not against it. We have to have answers. My point is, getting the answer is part of the *process*; it's not the product. The product is the knowledge that kids walk away with. The answers are just part of the process, and wrong answers, too. **[Slide: Wrong Answers]** One of the things you see—they're also part of the process, because—the main material you see in a Japanese or Singaporean lesson is the students' thinking; that's what they're teaching. They're teaching the kids how to think mathematically, and the starting point of most of the lessons is the student thinking in response to the problem posed, whatever it is. So the starting point depends on the students, and I'll come back to that.

So one of the things you'll see a lot of is the teacher saying, "Kim discovered a way of thinking that didn't work. Kim, explain your discovery to the rest of the class. Class, let's figure out why Kim's way of thinking didn't work. Kim, any ideas? Class, let's figure this out." And then they go on to figure out why Kim's way of thinking doesn't work. And Akihiko explained this to me, what this is all about. He said, "Here in America. . ." Akihiko, by the way, if you don't know who he is, teaches half-time at DePaul in Chicago and half-time in Tokyo, where he is revered as one of the main gurus of mathematics education. And so he knows both systems; he works in schools in both countries, and so he's really a gold mine of insight.

So I asked him what's going on with this, and he said, "In the U.S., you think there's something wrong with Kim, and you want to differentiate her, and so you find other kids who got that problem wrong, and you put them with Kim, and then you differentiate them." And he said, "I've been studying it for years. I still don't know what you're doing to someone when you differentiate them. And what you're seeing in Asia is quite different. We don't think there's something wrong with Kim. We see a way of thinking that Kim discovered, that any one of our students might think that way next week, or next year, or next month. You don't know which students are going to think that way, and you want all your students to understand why that way of thinking doesn't work. To understand why it doesn't work is to understand mathematics. The job is to understand mathematics, not to get the answer." So this is an asset—the discovery that a way of thinking doesn't work. It's not something that needs to be fixed; it needs to be exploited.

[Slide: Three Responses to a Math Problem] All right, so these three responses to a math problem: answer-getting is good. Making sense of the problem situation, that's very important; that's the main reason math is useful, that prepares you for work and life. Number three is what I'm talking about, because this is where the U.S. is deficient. We're okay on one and two; it's number three where we have a problem. **[Slide: Answers are a black hole]** But this answer-getting behavior is a black hole, and it's a bad habit; it's hard to escape the pull. So if I'm teaching, and I'm trying to get my kids to understand mathematics, they're going to say to me—they're going to push back, "Just show us how to do it. I don't want to understand it." Like they'll say what my own daughter told me, "Daddy, I don't have time to understand it; I just want to get it right on the test." So this is going to feel like breaking a habit.

[Slide: Answer getting vs. learning mathematics] So why are they different, these two different goals—answer-getting versus understanding the math? As a teacher, as teachers in the U.S. think—and parents and kids, by the way, and principals—how can I teach my kids to get the answer to this problem? If that's what they think, then I'm facing kids who are all over the place in terms of prior knowledge. Some know the algebra already, some don't know the multiplication table yet. So let's see, I could teach them math they don't know already, and then I can show them how to use the math they just learned, if they just learned it, to solve these new kind of problems. This looks like a classroom management nightmare. I have a better idea: I'll teach them how to get the right answer using math they already know, and maybe a trick or two.

Japanese teacher asks a different question: How can I use this problem to teach the mathematics of this unit? So when you look at a Japanese lesson, typically, a problem is presented, the answer comes out in the class discussion within five minutes or so, depending on the problem, and they spend another 20 minutes or half an hour on that problem *after* they have the answer. The U.S.—as soon as the right answer is out, boom, on to the next problem. What are they doing after the answer? Answer: mathematics. They're learning the mathematics.

[Slide: Butterfly method] Example: butterfly method for teaching adding fractions with unlike denominators. Something like this is the most common method used in the U.S. Here's how it works. **[Slide: diagram]** You draw the butterfly, which has two wings and a body. We're going to wind up multiplying on the wings, but first we put a plus sign at the antenna to remind us, and a multiplication sign on the tail, so now it's multiply on the wings: 4 times 1 is 4, I write it down; 3 times 3 is 9, I write it down. The plus sign reminds me, 9 plus 4 is 13, I write it in the head; 4 times 3 is 12; I write it in the tail. Answer: 13/12ths; terrific. But what does this have to do with mathematics? This has nothing to do with mathematics; it's an answer-getting method. When you add this to the curriculum, you have made the curriculum wider and shallower. This is where a mile wide, inch deep comes from, and this is why I'm saying that the focus on answer-getting instead of learning mathematics is one of the main causes of what's wrong with the U.S. curriculum—too many nonmathematical methods for getting answers to problems. Even though they give you the right answer, this is not the foundation of algebra; this is the foundation of confusion.

[Slide: Use butterflies on this TIMSS item] This was the easiest unlike denominator problem on TIMSS, which is the test for comparing different countries. And if you try to use the butterfly method, this is going to be pretty hard. If you use equivalent fractions—6/12ths plus 4/12ths plus 3/12ths = 13/12ths—it's really pretty easy. There's other examples. **[Slide: set up and cross multiply]** Set up and cross multiply; set up a proportion and cross multiply. This is correct; there's nothing incorrect about it. It's confusing. We get kids who are just finished three years of being confused by fractions. They've learned to invert and multiply, to divide fractions; they've learned the butterfly method to add fractions. And then we show them this equation that has the fraction on one side, an equal sign, and a fraction on the other side, and they think, "Oh, it's an equation," and we tell them, "No, it's not an equation, it's a proportion. And you're not going to solve the equation; those aren't fractions, those are ratios." This is. . .why are we doing this? The kids think they must have been absent. It *is* an equation, those *are* fractions; we have expressed the ratios as fractions, and we can use everything we learned about fractions to solve the equation. So we should tell the kids to set up an equation and to solve it; solve it using what you know already about fractions, and the basic tools of algebra you've already learned, like multiply both sides by a number, divide both sides by a number. Then you're using the time to prepare them for algebra, to get them progressing into algebra, rather than going down a cul-de-sac, a dead end, or some method for getting the right answers to the problems in this chapter that doesn't build a foundation for anything. And the criterion—remember, if my goal is to teach them how to get the answers, set up and cross multiply is very efficient. If my goal is to teach them mathematics and have them on their way into algebra, then it's a detour. Again, making it shallower and wider. The foil method in algebra; I'm sure you all remember it.

[Slide: Three major design principles] Okay, so when we set out to do these standards—focus, coherence, rigor—those were our three goals. **[Slide: Grain size is a major issue]** To get focus, let's look at some of the things that—focus and coherence—some of the things that matter. The states told us, make all the standards the same grain size, make them all the same size. This seemed very reasonable, so we tried to do it. It turned out to be torturous. Eventually we realized, this is wrong. The standards should be the same size as the mathematical knowledge the standard is about, and if they weren't—it's not all the same grain size, but this made us very sensitive to grain size of mathematics, of the knowledge the kids are supposed to acquire. And one of the things we discovered is, mathematics does not break down into lesson-sized pieces. This has a direct implication for how districts manage instruction in mathematics; how they manage content especially. So let me put it another way—there is no standard that can be taught in one lesson, and there's no lesson that is about just one standard. Put the standard up on the board you're teaching today? No, that's nonsense. Every lesson in a chapter is about the same cluster of standards. So if you want to put anything up there, put the whole cluster up there, and leave it up there all chapter long. We *did* discover that mathematics does break into chapter- or unit-sized pieces.

So the take-away for district leadership in mathematics, for me, on this, is spend more time planning chapters and units, and less time planning lessons. We have focused too much on individual lessons, and not enough on chapters. The answer to the question, "What's the math we want the kids to walk away from this chapter with?" is much easier to answer intelligently.

It provides a target that's much clearer to focus on for teachers and kids, than the question, "What's the math we want them to walk away from this lesson with?" Think chapters, don't think lessons, when you're designing systems to manage content in mathematics. So that's an important, practical take-away**.**

[Slide: What mathematics do we want students to walk away with . . .] Okay, what—so one of the things, for example, if you have PD, the PD should be about chapters if it's going to have math content, not just about how to teach a lesson; but it should also be about how to progress through a chapter. If you have departmental meetings or grade-level groups, they should talk more about the chapters they're teaching, and less about the lessons they're teaching. Lessons are important, of course, but in terms of managing the progression through content, it's much more sensible to do it at the chapter level; it's much more coherent. In fact, many books, the first part of the chapter is usually review of some kind. And I've worked with school districts where we start, not at the beginning of the chapter but some lessons in, and that makes more time available, and you use those early lessons as a resource.

[Slide: The importance of focus] How does covering topics relate to performance on the test?

LORI VAN HOUTEN

Phil, could I ask you a couple of questions?

PHIL DARO

This is from a study by Ginsburg et al. in 2005. They looked at the percent of the topics on the test that were covered by a country, because the TIMSS test had more topics than any country taught. So the percent taught—the country that taught the highest percent was the U.S., and they taught over 80% of the topics. The country that scored the highest was Hong Kong, and they taught about 50% of the topics. So a country that taught fewer topics that were on the test outperformed the country that taught the most topics, by quite a bit. So what translates into test performance is learning, not topic covering. I'll give you another bottom line: covering topics is a synonym for mile wide, inch deep. Much too much emphasis in the U.S. on managing the covering of topics, sending messages to the teacher that your job is to cover topics. What's a pacing plan if it isn't a message to the teacher about covering topics? We have to get away from defining the teacher's job as covering topics, and redefining it as teaching; in other words, the kids *learning*. Okay, so I'm gonna go just a couple slides and break for questions.

LORI VAN HOUTEN

This is Lori.

PHIL DARO

I'm not sure how I'm going to get the questions.

LORI VAN HOUTEN

You are going to hear me ask them.

PHIL DARO

Okay.

LORI VAN HOUTEN

So you talked—there are a couple questions that came in very specifically about the standards, and since we were on the topic of topics, can you describe how many of the topics that you removed from the standards that California put back in?

PHIL DARO

Okay. So first a little background—historical catching up. So each state was allowed to put 15—was allowed to add 15%, and most states added nothing or very little. California made the largest amendment of any state in the country. And specifically what they did is, in elementary school they added things, which we liked, and we only took them out to make it more roomier, so they put in more stuff. They put stuff back in about patterns, and a little more stuff about data, but not a lot. So I would say what they did in elementary school was okay; it was fine. The problem was the commitment California had to all students taking algebra in eighth grade. I could do a whole three hours on that. But California, which, remember, is scoring third, fourth, fifth from the bottom on NAEP, and has been there for 15 years, decided it should have higher standards than any other state, which it does. And incidentally, on that point, one of the things we saw in high-performing countries was not higher standards. What we saw was higher performance, and standards that were close to the performance. And so they deliberately would lower standards to get their standards closer to their kids. They were keeping their standards and their kids close together, just as any well-managed business will set goals that are high but within reach; that's exactly what they were doing.

California has had this tendency of very lofty standards, and . . . but very low performance; so the standards were rhetoric, in effect. So in that spirit they—this is what the original Common Core had. In eighth we had. . .at the beginning of eighth grade, you would start algebra; you would get through linear equations and systems of linear equations. The rest of algebra, which included polynomials, and quadratics, and all the stuff with exponents, didn't happen until high school. So we took the traditional American Algebra I course, which when you look at other countries, all the other countries, it's about two years of content. We made it two years. We did what the other countries were doing. In Singapore, in Japan; the most typical, in fact, in the world is to spread what we call Algebra I over grades seven, eight, and nine. So it's very misleading what shows up in the U.S. press. The study of quadratics, in fact, is spread over grades seven, eight, and nine, and if you go to the Singapore website, you can see their syllabus, and you'll find it. They're not doing the quadratic formula until ninth grade, and the Singaporeans are six months older in ninth grade than our ninth graders. So earlier is not higher, *higher* is higher; leaning more is higher. So California took all that high school part of Algebra I and crammed it into an eighth-grade course that was already full, because we had some substantial geometry in it as well. They moved a little bit of difficult probability down into seventh grade, and seventh grade was also already full. And so now you have these two years—seventh and eighth grade in the California version—that were just congested, and in particular they're congested with difficult stuff. So this is designed to defeat kids and teachers alike, because why would California do it? Maybe because they thought they had a high concentration of mathematically brilliant teachers in seventh and eighth grade; I don't know. But I think it was—they were solving a political problem. So now you know how I feel about it.

LORI VAN HOUTEN

Okay. Let me ask you about another question then, Phil.

PHIL DARO

All right, well, let me just finish, because that's not the end of the story.

LORI VAN HOUTEN

Okay.

PHIL DARO

So the legislature, not too long ago, passed a law, and the governor signed it, to fix this problem, and a commission is working on it now. And they are going almost certainly to do what all the other states have done, and spread algebra out over two years, eighth and ninth grade. And so this problem is being solved, even as we speak, for California, because the goal should be that you want all students to be college ready and college eligible, which has nothing to do with eighth grade algebra. It has to do with finishing the third year of mathematics, something sort of like Algebra II, before you graduate from high school. And so that's being cleared up. So now when we're done, California will look like almost all the other states and be much more reasonable.

LORI VAN HOUTEN

So another question that came in was, should eighth grade Algebra I teachers be preparing to teach the Common Core eighth grade standards, and I hear that your answer will—is, it will be yes.

PHIL DARO

Yes.

LORI VAN HOUTEN

Okay.

PHIL DARO

And bear in mind that eighth grade Common Core content—the majority of it *is* from Algebra I, but it doesn't get into polynomials and quadratics. And the reason for that is, we want more time per topic. This is the fundamentals of algebra; we don't want to rush through it. We want to take our time and go deeper.

LORI VAN HOUTEN

So here is another question that came through, then, that a lot of teachers are concerned about deleting topics and going deeper into other topics, because they're still being tested on those older standards. So how do you suggest that we address this concern? A similar question came through in one of the emails: What can I take out to make room for new things?

PHIL DARO

Right.

LORI VAN HOUTEN

Or to be able to go deeper. What can I eliminate?

PHIL DARO

Yeah. So the, you know, the bottom line advice I give is, look forward, not back. Use the Common Core to tell you what to keep in, and if it's not in the Common Core, drop it. Now that's going to cause some discrepancies with the CST. But not as much as most people would think. The alignment of any state's test to its standards is much weaker than, you know, common belief. But that said . . . so I'll just give an example: adding fractions with unlike denominators. On the CST, they're just going to be fraction problems. So if you dropped what California said, and replaced it with what the Common Core said, the kids would still get them right, if they're getting them right. In fact, a higher percentage of kids would probably get them right. But there are some things taught earlier in California in fourth grade, in third grade—the standard algorithms, for example, are taught earlier in California than in the Common Core. However, multiple choice tests cannot test *how* you got the answer, so that's not really that relevant; it's not going to make that much difference. They don't know if you used the standard algorithm or not on the test, because it's multiple choice. All they know is, did you get the right answer. So there are places, though, where there are topics in a grade level which. . .there's a discrepancy. There's not that many, and it would only make a difference if most of your students were getting those topics right, and you stop teaching it and then they got them wrong. And even that isn't that likely. It's not that likely that, you know, one of the things I tell teachers—how you're, if you've got 80% or more of your students getting a topic right and you think about dropping the topic, you've got something to lose. But if only 30% of them are getting it right, you don't have anything to lose by dropping the topic; they're not learning it anyway. So it's not as—it's much fuzzier and messier than people think, and there is less, you know, it's not going to be as damaging as you're afraid of, although there's some grade levels where there may be a hit. But still, in the end you're going to be better off looking forward than looking back. And don't try to do both. Remember, we're already teaching too much, so if you try to do both, it makes things worse instead of better.

LORI VAN HOUTEN

Okay. I'm going to ask one quick question. We have about a half hour left of our time together. So is there—and then I'll let you go back and finish up your webinar before we come back for a few more questions hopefully at the end. Is there a location, someone asked, where you can find out the rationale of why specific standards are placed at specific grade levels? That may help folks eliminate topics, too.

PHIL DARO

Yeah, one of the great resources is at the Institute for Mathematics and Education at the University of Arizona. There's Common Core tools, I think it's called. If you Google that and you look for it, and you put in the word *progressions*. There are progressions that were written by the authors of the standards and the people who wrote progressions for them while we were writing the standards. So when we were writing them, we asked experts of various sorts to advise us by writing a progression of topics. And then we used those progressions to build the standards. And those progressions were so valuable to us that we had them edited and rewritten so that other people could use them, and those are available on that site. And they explain why things are where they are. It's not always a question of, when should you teach this, fourth or fifth grade? The way we looked at it was, what are the foundations for this? And putting all of the bricks in to build the foundation for it took a certain amount of time, and it might be that your kids could learn it in fourth grade, but we didn't have all the bricks in place yet. So we put it—after we built the foundation, that's when we put it. And so that's why, in elementary particularly, sometimes you'll see topics coming later than they had been. So we took the time to build a solid foundation, instead of teaching it before they knew what was going on.

LORI VAN HOUTEN

Great, thank you, Phil. Why don't you go back to your presentation. I'll save these last couple questions that have come through, and if we have time at the end, we'll cover them then.

PHIL DARO

Okay. So close your eyes for a second, folks, and I'm going to skip ahead to personalization—let's skip this fractions stuff.

[Informal talk and skipping through several slides]

[Slide: Personalization] So there's a tension between personalization, which is about the uniqueness of each person, and standard, which is about the sameness of the curriculum. And that tension can be productive or it can be destructive, and I want to dig in, because I think we have a wrong view of it in our country. And just before I start, I want to remind us that, why do we have standards in the first place, if the social justice goals are the whole point of having the standards? So if we're not making them work for social justice, they're not serving their main purpose. And we've seen, even over the last 30 years with all our problems, a raising of the floor. The evidence from NAEP and other data is that our kids are doing better than they ever have. Every decade has been better than the previous, and every subgroup is doing better than it ever has before. So we have made progress, and we shouldn't forget that. And a lot of what I'm saying, since I'm talking about the changes that we need to make, imply kind of a criticism, but before going any further I just want to remind us, we've made important progress. According to NAEP, today's eighth graders are almost two grade levels ahead of eighth graders in the 80s. Latino students today are performing at the same level as white students performed in 1990. So we're making headway, but we just have to fix a few more things.

[Slide: Standards are a peculiar genre] So standards are a peculiar genre, and when you write them, you write them as though the kids learned approximately 100% of what preceded. But this is not true; kids haven't learned 100%, they haven't learned close to that, and that's not true anywhere in the world. The variety among the students is a fact of teaching—not just math, but every subject. And so the use of standards is not meant to blind us to the variation among the kids. There's no way to blind the teachers; that variation sits there in front of them. However, a lot of the instructional strategies that we put in place in a standards-based system have tended to ignore those differences or treat those differences as breaks in the system. But when you go to Japan and Singapore, you see their lessons are designed for variety; they assume it will be there. So how does that happen? Well, I've—it looks like my numbering these slides is a little off, so you'll have to adjust the numbering.

[Slide: Four levels of learning] So there are four levels of learning; this is the way I summarize what I saw in my visits to Japan and Singapore. The highest level of learning is, you understand it well enough to explain it to others, and as teachers, we've all had that experience of, you really don't understand it until you've taught it. And it is true that in the process of explaining—the process of explaining it to others—you have a much more complete and disciplined and detailed understanding than when you're just, just talking to yourself. The next level down is, you've learned it well enough so that the next related concept comes, you can keep up. You maybe don't understand it well enough to explain it to someone, but when you get the next thing, you've got it good enough so you can just keep going.

The next level down is, you can get the answers to this week's problems. That's not going to be good enough to learn future topics; that's not enough of a foundation for future topics. But it's better than the lowest level, which is just noise. So what's the implication of these four levels? This is about differentiation, personalization, the differences among kids.

[Slide: Four levels of learning: the truth is triage] The truth is that every day teachers have to go through a *triage,* and we should be honest about it. When you decide how fast to go, it'll be just right for some, too fast for some, too slow for some. When you decide how many problems to assign, which problems to assign, what's on the test—all of those decisions—who to call on, what answer to accept. All of those decisions are *triage* decisions. And our goal is to have a smart triage where all the students prosper, not—our goal is not to pretend everybody's learning the same thing, when we know they're not.

So what's, what is. . .you know, if you try to teach—one of the Japanese teachers explained, "When I look at my class, I see a field of rice. And I teach what I'm trying to teach, and after a little while I've harvested one row of students. They understand it. They come and join me and help me teach the rest of the students. And after a little while, I have another row, and another row, and towards the end I have most of the students helping me teach the few remaining students who haven't learned it yet. But still I have to go on. I don't always get to the last students; but every day that's what we set out to do, the whole class." Akihiko explained this as, "Our pedagogy begins with the common humanity in the classroom, and the differences are on the margin, and our goal is for the *class* to understand it." And he said, "The problem in America is, you start with the differences, and then you exaggerate them, and you don't take advantage of the common humanity, which is the way you are there to solve it."

So how do I know when to move on? So using these four levels, I want at least a third of my kids to understand it well enough to explain to others, and I want that because I'm going to use social processes in my classroom—think/pair/share, peer pairing kids up. I love pairing because everybody's listening or talking; no place to hide. Trios; I try to keep the groups small. And if I have a third of my kids who understand it enough to explain it to others, I've got embedded tutoring going on. And I have it going on, not as tutoring, but as a natural process in the class, and if I'm not rushing through, there's time for that tutoring to take place. So I, you know, if I get a third, then this social process, the embedded tutoring, the think/pair/share kinds of stuff, will take care of most of the rest. And by the way, when your students who have caught on quickly—your brightest, most ambitious students—already know how to get the answer, what are they doing when they are trying to explain it to others? What they're doing is learning it more deeply. They're not wasting their time; they're going deeper and deeper into the mathematics. Mathematics has infinite depths and the more—if I'm trying to explain it to someone who's having a hard time understanding it, I'm going deeper and deeper into the mathematics.

Okay. So let's go to the next level. But I want most of the rest of my students to learn it well enough so when I start the next unit, they've got a good enough foundation to stay with it. Still it will happen, at the end of some days there are some kids I just can't get there, and I just have to move on, because this lesson I wanted it to go one day, but already it's two days. And yet would I settle if I was a real teacher with real kids, for getting the answers? Honest answer, yes. I know I would settle for that, because you just can't wait for everybody every day. But I won't be happy that I'm settling for it. I'm certainly not going to aim for it; I'm aiming for number one: well enough to explain it to others.

[Slide: efficiency of embedded peer tutoring is necessary] So within the same topic we'll get different levels of learning. Some kids will learn it deeply by explaining, and they'll be an asset to the others; some kids will learn it well enough to keep the momentum going forward. The kids who only could get the answers, I think they need tutoring after school, and they can profit from that tutoring. All right. I'm going to skip that [slides]. **[Slide: When the content of the lesson is dependent on prior mathematics knowledge]** So I want to—the implications of this—remember, this all starts from the idea of how do you use standards in the classroom when kids are different? The implications of this for lesson design are important. Many districts have used the I-we-you design, and I asked about this, because it was attributed to Singapore, and in Singapore they knew it well. And they said they do use it sometimes, but not as often as the opposite. I said, "What do you mean, the opposite?" And they said, "The opposite is the you-we-I design." So I asked for explanation, and that's what I'm going to show you.

The problem with I-we-you design is that it ignores prior knowledge, and so it's well suited for content that doesn't depend on prior knowledge. And so there's two situations where that's the case: one is when you're introducing new content that's not really related much to prior knowledge, which happens; the further you go, the less it happens. And the other situation is where you've been working in a topic for a week or some time like that—three, four, five, six days—and you've got the knowledge in that topic to a point where all of the kids are more or less close together. And at that point an I-we-you lesson, to consolidate that knowledge and extend it, is a good idea. But much of the time you don't have that common starting place.

[Slide: Minimum variety of prior knowledge in every classroom—I-we-you #1] So here's the assumption for an I-we-you lesson. The lesson has a start; the kids are more or less in the same place; you plan activities which take that amount of time to get from the red to the green line; and you can get the kids there**. [Slide: Variety of prior knowledge in every classroom: I-we-you]** But, if the kids are all over the place, as they will be on many days, the majority of the days—in Singapore, they said about two-thirds of the lessons—the time kids need is much more for some kids than the planned time, and less for some kids. So what actually happens when you try an I-we-you lesson with, you know, a scheduled lesson plan, **[Slide: I-we-you #2]** what happens is, the kids don't get to the target level; they crash. Only two kids in this example would get there out of five. And the teacher can't really tolerate this, because this is classroom management disaster. So they have to adapt, and what do they do? **[Slide: I-we-you #3]** They resort to answer-getting. Answer-getting enables everybody to come to an emotionally satisfactory landing place: I got the answer. And the downside of this is, even students B and E, who could have gotten to the target, don't get there because the target wasn't "understand it well enough to explain to someone else." The target was "get the answer." So we've lowered the standard; when we lower the standard to answer-getting, even our best students are cheated.

[Slide: You-we-I] So here is the typical lesson; let's just call this the differentiated lesson, or the you-we-I lesson. The you-we-I means we start with *your* thinking—each of your student's different thinking—we listen to each other and understand each other's thinking, and then I as the teacher pull us toward the grade-level way of thinking. And so in this case, the starting point is a wavy line. The students think about the problem in different ways, and I'll give an example: if you give our problem of Jason—how long did it take Jason to run 200 meters, or whatever it was, running 4.5, four yards in 4.5 seconds. A lot of kids will solve that problem, sixth-grade problem, by skip counting, essentially: 4.5 seconds in 40 meters, and 40 yards in 4.5, 80 yards in 9 seconds, 160 yards in 18 seconds, and 40 more yards, that's 28. So that way of thinking gets you the answer, but that's essentially a second-grade way of thinking; that's what student A might have done. Student C might have made a table; you know, 4.5 is 40, 9 is 80, etc., like that. That's a fourth-grade way of thinking. What you want is the unit rate—how fast is the student going, how fast is Jason going—and that's what student B might have done. And so the way you handle the class is: everybody listens to student A, everybody listens to student C, and when student C is doing the table, you make sure student A understands the table. And then student B explains it, and you identify the correspondences between student B's way of doing it at grade level, and how student A and C did it. And that's the typical kind of lesson.

So this is differentiation that'll work most days for most kids, not all kids. And it's the first response for dealing with differences in the classroom. Of course, it's not going to work every day, and it's not going to work for all kids, and we still need interventions and so on. We just need to minimize the amount of intervention. **[Slide: Differences among students]** All right, so I hope that made you think a little bit about how standards relate to differences among students, and given the time, I think I'm going to stop here and see what other questions we have.

LORI VAN HOUTEN

And we do have them. Lots of questions came in beforehand, and now, about instructional practice, and since that was sort of the topic we were just addressing, one question was, "Would use of Singapore Math cover our Common Core State Standards and be appropriate?"

[Flipping through slides; Slide: Mathematical Practice Standards]

PHIL DARO

Singapore Math is an American product which is an adaptation of a Singapore book that was for sale there some time ago, and as I understand it, not a very big seller. There are—the current

Singapore textbooks that are in use in Singapore are written in English, and they can be purchased off the web or directly from Singapore, and they're not very expensive, and they have teacher material with them. If you're interested in doing what they're doing in Singapore, you should get what they're using in Singapore. Also, and I like these even better, there is . . . the Japanese textbooks have been translated into English from K through grade 9 and they—you can get those through Global Ed Resources. And I think those are such lovely books; I have no business connection to them whatsoever. In fact, I do have a business connection to Pearson, so they'd be probably pretty cranky that I was suggesting someone else's product. But I do think every teacher should have those Japanese books, even if they're not used with the kids, and they'll get plenty of resources from those books. So the question was, does Singapore—you'd still have some work to do. Singapore Math is probably as well aligned as anything else we have here, but I haven't seen a real breakdown of it. No matter what you're using, you'll have to do some aligning.

LORI VAN HOUTEN

Okay, I'm going to switch topics a little bit here. Today, and before the webinar, some questions came in around high schools in particular. The one that came through on the webinar was, "Please share how math AP classes compare with Common Core Standards," but also, ones that came through before were more about, "What are your feelings about what's most appropriate for high school math, traditional or integrated, and provide the reasons or support for your choice," so math AP classes currently . . .

PHIL DARO

See, Common Core went to. . .I'm sorry [for interrupting]. . .

LORI VAN HOUTEN

Go ahead.

PHIL DARO

Okay. Common Core only went to college ready; we didn't go beyond college ready. So we did not write standards for pre-calculus and calculus. And if you go to the UC website, to admissions, and look at the A to G requirements, you'll see that UC has already changed the math requirements to fit the Common Core, as most universities in the country have. So UC is already using the Common Core for its admission requirements. How do I feel about AP? Well, it's more important to understand the fundamental stuff deeply than it is to rush ahead to greater topics or advanced topics. The problem is, admissions awards extra points for AP courses, and so you can't advise kids not to get those extra points for admissions purposes. So it's up to the colleges to change the admissions treatment of AP courses.

LORI VAN HOUTEN

Does that impact your thinking about whether a traditional model or an integrated model is more appropriate at high school?

PHIL DARO

No, they both have the same relationship to AP, because AP comes after them, and both of them lined up in the same place. So in the Common Core, we did not write courses at high school. The states wanted to decide, and most states are allowing either, making it a district decision. My own view is, we're the only country that does not teach algebra every year. Every other country teaches algebra every year, and in order to teach algebra every year, they spread geometry out over, and more or less teach geometry every year as well. Whether algebra and geometry are well integrated or not varies. So I think the traditional algebra—take a year off from algebra and do geometry, and then start up algebra again—really doesn't make sense; no one else does it and we have to get over it. The main opposition to getting over it is not people who are enthusiastic about it, but people who think the devil you know is better than the devil you don't. You're afraid if you lose the traditional sequence the baby will go out with the bath water. So that's a legitimate fear, but one way or another, my own feeling is we've got to get more in line with what the other countries have done. When we do that—I would not. . .if I was a school district, I would not force them. If most of my teachers wanted to stick to traditional, I would stay with them. It's not something you want to jam down against the teachers' will; that's my own view.

LORI VAN HOUTEN

So without instructional materials, what should school districts be doing now, and who's doing it well?

PHIL DARO

So one good thing about Common Core is we don't have to limit our work to California, because we're all using the same standards now, with the exception of Alaska and Texas and Virginia. So there is . . . New York City school district has gotten a lot of resources and has used them pretty wisely, and their website is a great resource for any school district. So that's one place to go to see a district doing things well. The state of Vermont, the state of Delaware, have also led down in front; their websites are useful, and also keep an eye on the Massachusetts website. They, those states have better budgets than California, and they've jumped on it in better, you know, better resource ways, so they're useful.

The mathematical practice standards which I have up there [slide]; there are eight of them, they're on pages 6 through 8 of the standards. I would say focus on those, those are K–12; those are everyday lesson-level issues that you want your mathematics classrooms focused on kids doing these things. These are kid—student practices, not teaching practices—and the assessments are trying in various ways to incorporate these in the assessments. This is the expertise we want kids to develop, and you don't get tangled up in the differences in content; whatever content you're using, these practices are important. So I'm urging current investments in PD to focus on the practices. All the publishers are both revising current things and writing new things, and if you have a product in hand, you should take advantage of whatever your publisher has done in terms of writing more detailed advice about which chapters and lessons go with which standards. And, of course, those are going to vary in quality. But I know that just from the questions we get, the authors from the variety of publishers, they have teams working on it. So I would again look forward to that stuff. And the digital resources are also growing, so the online stuff is all going to be tagged to the Common Core. And there's more and more initiatives going on related to that, so keep an eye on the digital resources.

MEG LIVINGSTON ASENSIO

Great. Phil, I think that's a good stopping point. Lots of food for thought on the webinar! I want to let everyone know that we will shortly be posting a recording of the webinar and the PowerPoint slides at relwest.wested.org, and finally, we would like everybody to take just a second, if they would, to fill out a survey, and I'm going to put that up now. The link is also in—well, that didn't work. The link is in your, in the chat box, and you can just do a copy/paste on that. I'm going to try this one more time; it's supposed to push it right out to you, but if you would not mind doing a copy/paste and filling out a short survey. One of the questions is, what topics you would like us to cover in future webinars, so . . .

LORI VAN HOUTEN

It worked, Meg; it's come up for me.

MEG LIVINGSTON ASENSIO

Okay. I'm not seeing it on my screen, so I hope others are seeing it on theirs.

PHIL DARO

I see it on mine.

MEG LIVINGSTON ASENSIO

Great, okay, well. You can just go ahead and fill it out right now online, or you can copy the link and fill it out later, but we really would appreciate your feedback on the webinar, as well as your suggestions for future topics that you would like us to cover. So I think that concludes the webinar. Thank you so much, Phil, for doing this, and we'll look forward to hearing from you again soon.

PHIL DARO

Okay, thank you.

MEG LIVINGSTON ASENSIO

Thanks everyone.